

*Mathematical Algorithm for Understanding Numbers and Their Operations Integrated
Into the Cornerstone of Learning in a Pelmanism Game: Mathesso*

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Abstract

Mathesso, a novel board game focusing on explaining elementary mathematics to kids of all ages, is presented. Using the principle „learning by playing,“ Mathesso allows children to acquire a rich intuition about the structure of the number system. The game has a strong potential for reaching comprehensive and rapid growth as a learning tool worldwide, including in developing countries, since there is no need for a good quality teacher to reach full game potential. The game's goal is to activate cognitive processes responsible for the origin and development of mathematical intuition. Mathesso is based on the palmesian principle connected to mathematical operations. The teaching principle is based on the unique mechanism of reverse synaesthesia responsible for perceiving dependencies between numbers. Using this method, children can gain a deeper understanding of inherent connections in math while avoiding mechanical memorization, which is the leading cause of difficulties in learning mathematics. By playing this game, kids can quickly and naturally understand the principle of multiplication, division, powers, prime numbers, Fibonacci numbers, factorials, and more, explaining their principles. Moreover, the game is also strategically very appealing and thus has the potential to become a challenging strategic game on an international competition level.

Keywords: Reverse Synaesthesia, Pelmanism, Mathematical Intuition, Numbers, Strategic Game

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Introduction

The game's goal is to activate cognitive processes (as described by Petty, R., & Briñol, P. (2015)) responsible for the origin and development of mathematical intuition (Johnson, S., & Steinerberger, S. (2019)). The Mathesso game is designed to influence children while playing subconsciously and uses basal arithmetic (Song, C., et al. (2021)), to introduce them to selected mathematical operations.

While playing Mathesso, pre-school children may quickly evolve the so-called backbone algorithmic system (Elizabeth S. Spelke (2017), Katherine D. Kinzler, & Elizabeth S. Spelke (2007)). Children do not need to know a single number, and simply by orienting themselves by colors, they understand the principle of multiplication, prime numbers, powers, and, among other things, they will gain intuitive knowledge of the multiplication table.

Once upon a time in the ancient history of humanity, there was a time when people did not know how to count (Osterlind, S. (2019)). They couldn't count the sheep in the pasture nor share a loaf of bread fairly. They couldn't tell how many children they had or if they had enough food. They couldn't even count the days left before it was spring again. Imagine them sitting in a freezing cabin with no more food and no idea how long the snow storm had lasted because the only variable they could work with was yesterday or tomorrow. Two days earlier was the same as last year for them. And all that because the math didn't exist yet. The first very tentative hints of mathematics began to appear in prehistoric times, but it wasn't born until ancient Greece (Narlikar, J. (2021) and Cooke, R. (2013;2012;2014)). It has been evolving ever since. Today it is everywhere in every object around us and every moment of our lives. Without mathematics, we couldn't build a house or make tea. We wouldn't have cars or planes or showers or carousels. Without maths, we wouldn't be able to make a phone call, and more seriously still, we wouldn't be able to watch cartoons. Everything you can imagine is imbued with mathematics. Sadly to say, many kids don't like mathematics despite the fact how important it is. They fear, don't understand, and never really learn. And then they miss it all their lives.

Why are children so happy to learn to talk and paint and write and read every puzzle and anything interesting, but not mathematics, which is the essence of interestingness? How is it that they don't see its intrinsic beauty, and despite all the efforts of parents and teachers and the positive and essential ways that math improves our lives, most people hate it (Larkin, K., & Jorgensen, R. (2015;2016), Xolocotzin, U. (2017) and Perschbacher, E. (2016))?

We found out how in many cases, young children develop the ability to perceive numbers and what causes them to stop getting on well with the numbers at some point. And how this misunderstanding turns into a lack of self-confidence, and how children gradually develop a kind of allergy to the numbers and then never want to hear about it again (DiStefano, M., O'Brien, B., Storozuk, et al. (2020) and Stearns, M. (2013)). Therefore we created a universal math game in which players don't have to count or even know the numbers. The game can solve these misunderstandings. The game that children and adults enjoy and that not only teaches them how to count but, above all, opens the way to fundamental mathematics. Because counting and mathematics are not the same (Ziegler, A. (2017)). This is the first of many misunderstandings set straight by playing Mathesso.

The most crucial educational mechanism of Mathesso is reverse synesthesia (Ashok S. Jansari, Mary Jane Spiller, & Steven Redfern (2006), Simner, J., & Bain, A. (2018), Green, J.

A., & Goswami, U. (2008), Gebuis, T., Nijboer, T. C. W., & Van der Smagt, M. J. (2009). Rinaldi, L., Smees, R., Alvarez, J., & Simner, J. (2020)). As complicated as it might sound, synesthesia itself is well known. It is a phenomenon when people associate certain sensory perceptions with others. For example, the ability to perceive music in such a way that colors or smells are perceived simultaneously as tones (Lacey, S., Martinez, M., McCormick, K., & Sathian, K. (2016), Tilot, A., Kucera, K., VINO, A., Asher, J., Baron-Cohen, S., & Fisher, S. (2018), Ward, J., Huckstep, B., & Tsakanikos, E. (2006) and Jäncke, L., Beeli, G., & Esslen, M. (2005)). Some of the greatest mathematical geniuses, who can count pi to one thousand two hundred and twenty-one decimal places off their heads, have this ability and use it. However, they obtained the knowledge strangely and unpredictably. “Well”, we said to ourselves. What if we used colors in some meaningful way instead of a hard-to-explain numerical system? For example, in the way that a different color represents each digit. This seemingly trivial substitution has the potential to open up a new level of teaching math.

We all had to memorize multiplication tables at a young age until we knew them by heart. The reason is that the origin of multiplication is empirical and not, as most people think, some clever formula. The absolute basis of mathematics, arithmetic, is a millennia-long observation of millions who have gradually discovered that if there are three people, they will never divide two sheep fairly. If there were four, they could get along. They would split into two groups of two, each group would take one sheep, and that is it. Peaceful solution. However, if there were five trying to split two sheep, a conflict would arise.

Indeed, the whole arithmetic is based on a set of observed basic properties of different situations. These observations form a multiplication table (Parker, G.. (2019), Barka, Z. (2017), Shaw, J.. (2022), and McElderry, H. (2021)). And that is the core of the problem, for there is no other way to acquire knowledge of this very long and tedious series of similar columns of easily interchangeable characters than to memorize it. Thus, it is no wonder that many children do not enjoy it. And if they don't want it, they may find it very difficult to learn. The younger they are, the harder it is (Spinillo, A., Lautert, S., & Borba, R. (2021), Nunes, T., & Bryant, P. (2021) and Cheng, Y.L., & Mix, K. (2014)).

Another problem is undue pressure to perform and the resulting low self-confidence. And most importantly, a way-too-early transition from arithmetic to mathematics is difficult to fully understand without perfect mastery of the basics. This creates the vicious circle results which we know so well (Farrés-Tarafa, M., Bande, D., Roldán-Merino, J., et al. (2021), Safitri, R., & Widjajanti, D. (2019) and Francis, B., Connolly, P., Archer, L., et al. (2017)).

Principle of the Mathesso game

We developed the idea of turning the entire arithmetic apparatus into a game using the pelmanism principle (Wilson, S., Darling, S., & Sykes, J. (2011)). Children are successful in this game due to their extraordinary memory, which they enjoy. The basic principle of Mathesso is the same – to find two jettons that are identical. No matter what the strange little squiggles on the top and bottom of the jettons mean (those squiggles represent numbers). Moreover, reverse synesthesia makes it as easy to imprint arithmetic into their mental system as easily as if they were learning to talk. Children will easily substitute numbers with colors using a unique color ruler and understand the numbers as a byproduct of playing the game. However, we do not have to tell them all of that. Nor do we need to burden them with the fact that by playing this game, they will create a backbone algorithmic system involving their subconscious knowledge of all numbers, multiplication, division, powers, prime numbers,

Fibonacci numbers, factorials, and others. Nor will we bother them with the existence of the phenomenon of zero and its specifics. We will just let them play.

this game results are the research of several years of research into the thinking, comprehension, and learning, and its effect has potential to invaluable despite its relative triviality. If the methodological procedures we have developed are followed, children may gain a fundamental understanding of arithmetic, deep mathematical intuition (Hipolito, I. (2015)), and procedural logic. Indeed, the game overlaps with number theory and game theory. Higher variants of Mathesso (the following section) require a constantly changing strategy depending on each player's score, the number of players, and their skill and strategy. Players can reveal one or two cards at their discretion in each turn. Indeed, in higher versions of Mathesso, considerations need to include both the probability that a player scores a pair and that turning a second jetton gives additional information to other players. Understanding the different strategies in higher versions becomes crucial and arises quite naturally.

Nonetheless, as a result, we remind ourselves that we are still emitting about playing a memory game. None of the described herein requires any mathematical knowledge. Playing the game comes with an intuitive understanding of probability calculus, strategic reasoning, risk fluctuation, and the ability for reflective calculations and parallel thinking (Yang, X., Zhao, G., Yan, X., Chao, Q., Zhao, X., Lu, T., & Dong, Y. (2021)). And most importantly, insight into how beautiful mathematics is and its beauty is simply no accident. Other bonuses from playing Mathesso are advanced concentration and the ability to orient in a large set of confusing information. Children also practice memory and fine motor skills. Even though Mathesso is specially designed to be played by preschoolers who have not yet learned to read and write, older children and adults may find it very mind-opening too. However, for the effects mentioned earlier to occur, methodical procedures must be followed, and, above all, the game needs to be practiced regularly. The more active aversions in mathematics, the more effort it will need to eliminate them. The methodology development for identifying blockages in mathematics will be included in the forthcoming research.

Instead of teaching multiplication tables, parents can spend time with their children playing Mathesso. They may teach their children and even themselves incomparably more in less time. In the beginning, playing lighter variants with fewer jettons (the following section) is advised, and not be ashamed to use the color ruler whenever you lose confidence. The fewer jettons left on the table, the more parallel calculations occur. If a player gets lost in the rules, he can always switch to a simple memory game and navigate himself by jetton position. If a player finds the game too complicated, he can reduce the number of jettons to play with. After you overcome the initial uncertainty and perseverance, you'll discover that Mathesso is comparable to a chess game in its complexity. The beauty of Mathesso lies in its logical structure, working at the basic level of reasoning. Two times four equals eight, and no more profound thought or analogy is behind it. Thus, subconsciously, the source code of logical thinking is being tuned, and its effectiveness increases significantly. Mathesso works in the simplest possible way with the most complex phenomenon. Thinking (Drukarch, B., Holland, H., Velichkov, et al. (2018), He, K. (2017), Baggs, E., & Chemero, A. (2020), O'Keefe, P., Horberg, E., Sabherwal, A., Ibasco, G., & Binti Zainal, A. (2021)).

One of the game strategies

In addition to the features described above, Mathesso can develop not only mathematical knowledge but also memory, strategic thinking, and perception of randomness to a great

extent. In this chapter, we will focus exclusively on the basics of where the development of the strategy may be heading.

In this game, a player will often find himself in a situation where he does not know or cannot remember even one face of jettons. It turns out that as early as the second game a player plays, he begins to think in these situations about which face number is best to flip to maximize the probability of successfully converting the second jettons.

As a first example, the initial state of the most miniature version of this game, namely Mathesso ATTO, is given for straightforward interpretation. From the graph, it can be seen (Figure 1) that it is most advantageous to flip the jettons with the number 1 (44%) followed by 5 (38%). In the game, this advantage is compensated by the fact that before the first turn, the players order is determined by flipping the jettons with 1, and the probability of the other numbers increases. Furthermore, the number 5 contains a prime number worth fewer points. Even this most miniature version of the game thus becomes balanced in terms of this strategy and opens up the possibility of using other techniques.

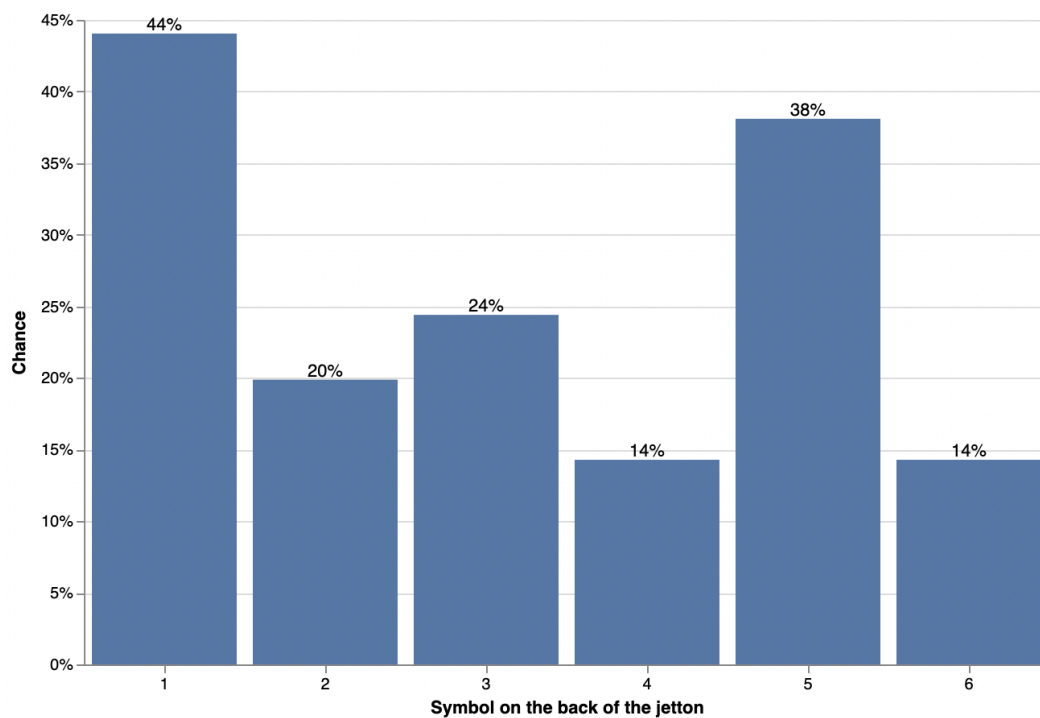


Figure 1: Probability of success when turning the first jetton according to the number on the white side of Mathessa ATTO jettons.

Another significant component is the probability of a successful move changes profoundly with each change. The following graph shows (Figure 2) how much the given probabilities can change when the previous player picks up only one pair of jettons. This shows that the game strategy can fundamentally change depending on which jettons are turned over initially.

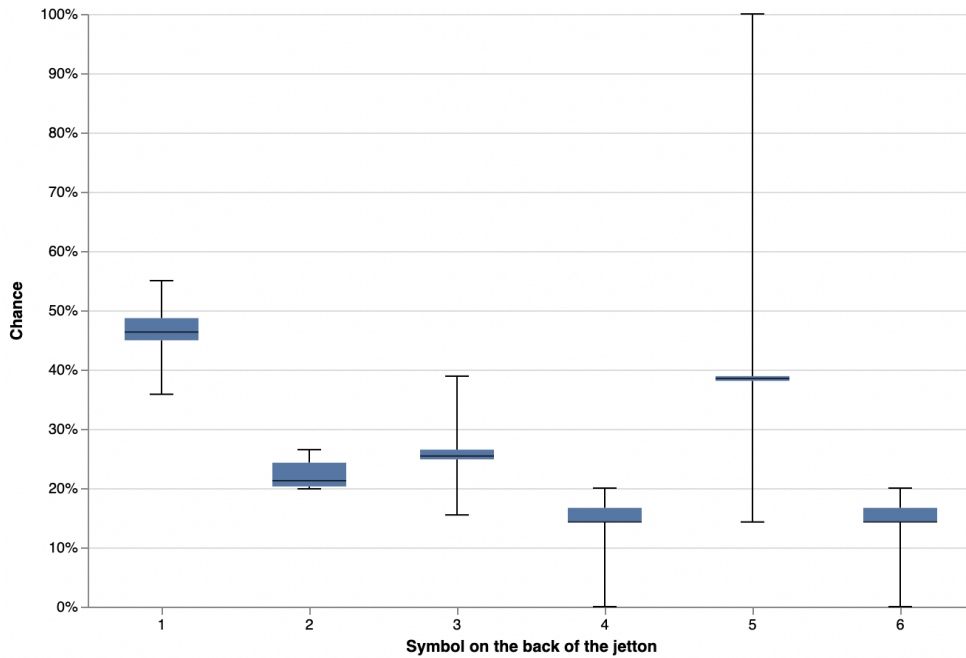


Figure 2: The Change of success when removing one pair from Mathesso ATTO.

This strategy is even more significant for Mathesso GIGA. As seen in (Figure 3), the knowledge of prime numbers, which are free to collect with this knowledge, pays off here. However, they are only for one point. On the other hand, factorial tokens (!) are worth 4 points, and they are represented with the lowest probability (6.2%) at the beginning of the game. Furthermore, the game is designed so that none of the most frequently occurring numbers have an advantage at the beginning of the game, as can be seen on the numbers 1 to 11, where the probability varies from 6.9% to 8.1%.

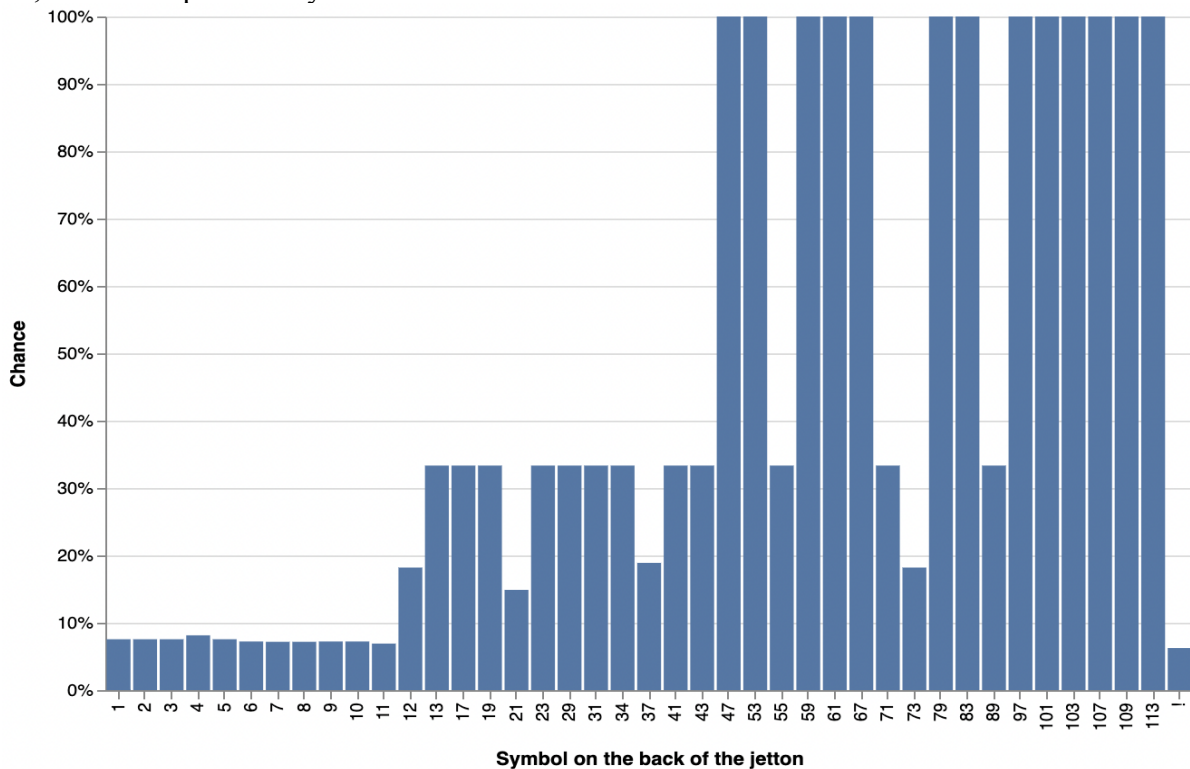


Figure 3: Probability of success when turning the first jetton according to the number on the white side from Mathesso GIGA.

Teaching method

Mathesso is a game for teaching mathematics to children from 3 years old, adults, and university students. Therefore, the game is designed to have the lowest possible entry barrier (Schmidt-Jones, C. (2021;2022)) and, simultaneously, contain a high level of complexity. It is, therefore, possible to eliminate pairs of tokens from the game, depending on which mathematical property the player is learning. There are also a set of rules that can be followed depending on the age or agreement of the players.

Mathesso can be played in the same way as "Pexeso", "Pairs", "Memory", and "Concentration Memory Game" or also known as the "Pelmenism system" (Wilson, S., Darling, S., & Sykes, J. (2011)). So the game doesn't need to be explained in many cases. The rules can be summed up in a few sentences: "The jettons are laid out white side up. Players turn over a pair of cards so the other players can see them. If the flipped jettons have the same picture (color and number), the player takes them. If the cards are not the same, he turns them over. The next player in the sequence continues in the same manner. It is played until all the cards are taken apart. The winner will be the player with the largest number of found pairs."

With the "Pexeso" rules and without knowing the meaning of the numbers, the players can begin to see the connections and turn the correct jettons after a few short games. So that the game dynamic where players know how to use a ruler or know the multiplication table, prime numbers, and other types of tokens is preserved. Pairs of jettons are rated with points according to the difficulty of finding and game complexity.

- 0 points for each pair from group zero (black background).
- 1 point for each pair from the prime number group (yellow background).
- 2 points for each pair from the group of Fibonacci numbers (Dunlap, R. (1997)) (orange background)
- 2 points for each pair from the group small multiplication table (colored on a gray background)
- 3 points for each pair from the power group (colored on a circular background)
- 4 points for each pair from the group of factorials (Mingarelli, A. B. (2013)) (rainbow background)
- 5 points for each pair from group CJV (yellow-blue background)

When learning the multiplication table, the student is often confused about the fact that the number 16 is 2 times 8 and at the same time 4 times 4. This confusion can lead to further misunderstandings in the student's further studies. This game turns this phenomenon into an advantage with the rule of multiplying results from pairs of jettons with the same number. In the given case, the player has 2 points for 2 times 8 and 3 points for the power of 4. Therefore, this combination's resulting number of points is $(2 + 3) * 2 = 10$ points. A student with this rule deliberately looks for such numbers and tries to pick them up. The opponent, on the other hand, tries to collect jettons with the same number.

One of the last recommended types of jettons is the "Cooper-Janečkova variety" (CJV) jettons. These tokens are the only ones that break the "pexeso" rules and are taken away if they only have the same background. At the same time, if the player picks up a combination of two, the jettons are worth 12 points, and the player has one extra turn at any time in the game. This jetton gives the game many possible strategies and even better game dynamics which will be described in detail in subsequent research and publication.

Conclusion

The new board game Mathesso is built on the pelmasian principle connected to mathematical operations. The reverse synaesthesia as a learning mechanism (Ashok S. Jansari, Mary Jane Spiller, & Steven Redfern (2006), Simner, J., & Bain, A. (2018), Green, J. A., & Goswami, U. (2008), Gebuis, T., Nijboer, T. C. W., & Van der Smagt, M. J. (2009). Rinaldi, L., Smees, R., Alvarez, J., & Simner, J. (2020)), a system of unique colors, is used to imprint mathematical dependencies in the memory. In fact, the player does not even need to know the number symbols in order to play. Thanks to this, it is possible to learn multiplication, division, powers, prime numbers, Fibonacci numbers, factorials, and more in a non-violent method, just by playing this game. The forthcoming research will deal with creating a methodology for defining improvement, not only in mathematics but in diverse groups of people, especially in children of preschool age. The game has been preliminary tested (played) on individuals and modified to its current form with the help of feedback. Preliminary results at elementary schools in the Czech Republic show that after learning Mathesso, pre-school or first-grade school kids have a significantly more warm relationship with mathematics. Fundamental division and multiplication skills have been demonstrated without additional learning. In several cases, the kids understood the meaning of prime numbers purely by playing the game. Aversion to mathematics can be resolved for adult players by playing the game. In addition, Mathesso has been preliminarily found to significantly improve memory, strategic thinking, or the perception of randomness for several individuals.

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